Ideological Innocence

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Abstract

Quine taught us the difference between a theory's ontology and its ideology. Ontology is the things a theory's quantifiers must range over if it is true, objects like people, chairs, incars, and Eiffel-Tower-Noses. Ideology is the primitive concepts that must be used to state the theory, things like quantifiers, operators, connectives, and predicates. This allows us to split the theoretical virtue of parsimony into two kinds: ontological parsimony and ideological parsimony. My goal is help illuminate the virtue of ideological parsimony by giving a criterion for ideological innocence - a rule for when additional ideology does not count against parsimony. I propose the expressive power innocence criterion: if the ideology of theory one is expressively equivalent to that of theory two, then neither is ideologically simpler than the other. In its favor I offer the argument from accuracy, showing that plausible rival criteria for ideological parsimony divide logical equivalents, and thus cannot be an account of a theoretical virtue that is supposed to make theories that have it more likely to be true than theories that do not. Next I consider its ramifications, eliminating rival views and passing judgment on some arguments from parsimony that can be found in the literature on first-order metaphysical questions. Finally, I consider two objections. First: I address an objection arising from the possibility of languages with a 'primitive' operator that allows us to list a theory's primitives in the object-language. Second: I address an objection raised by Nelson Goodman against attempts to reckon simplicity by expressive power. Both objections fail. I conclude that the expressive power innocence criterion is true.

1 Introduction

Quine taught us that theoretical commitments come in two varieties: ontology and ideology.[1] A theory's ontology is the entities that must exist if the theory is true: things like chairs, gods, electrons, incars, and Eiffel-tower-noses. A theory's ideology is the primitive notions or concepts employed in its most perspicuous statement: things like quantifiers, operators, predicates, and connectives. Good theories minimize their commitments. Better theories minimize their commitments more. This minimization of commitments is typically codified in the theoretical virtue of parsimony, which may be split into two components: ontological parsimony and ideological parsimony. My interest here is in ideological parsimony, There is, unfortunately, no widely accepted theory of what ideological parsimony amounts to. I have no such theory on offer. But I do wish to defend a sufficient condition for when additional ideology does not offend against parsimony; a criterion of ideological innocence, so to speak. I will argue that when adding ideology does not increase expressive power, it does not count against a theory's parsimony.

In defense of my proposal I offer what I call the argument from accuracy. The argument takes a specific conception of how epistemic theoretical virtues do their job and combines it with a theory of epistemic reasons to show that any adequate analysis of ideological parsimony must include my expressive power innocence criterion.

After giving the argument from accuracy, I explore the consequences of my proposal for the literature. First, I show that other proposed approaches to ideological parsimony fall to the argument from accuracy.

[1] Quine [1951], [1983].

Then I consider several instances of parsimony arguments in first-order metaphysical debates, and show which my proposal deems failures.

Finally, I will consider two objections. The first comes from the use of languages containing a 'primitive' operator, which allows us to supplement theories with an axiom listing their primitives. The second, from Nelson Goodman, aims to show that all approaches to parsimony based on expressive power fail, because they remove the motivation to replace primitives with defined concepts. I argue that both fail.

2 The Expressive Power Innocence Criterion

Expressive power is a property of languages. For our purposes, we can think of a language as a set of symbols plus some formation rules. Roughly, a language's expressive power is the range of things it can be used to communicate. Two languages are expressively equivalent when they can communicate the same things. A bit more precisely: two languages are expressively equivalent when there exists a one to one map from sentences of the first to sentences of the second such that the sentences paired up are true in all and only the same models, which for our purposes we can think of as mathematical structures that together with a semantics give the meaning and truth conditions of the sentences of a language.[2] We can think of a theory as a set of sentences in some language.

We can compare the expressive power of ideologies by comparing the expressive power of languages whose syntax includes only symbols representing their primitives and the things that can be defined out of them (with perhaps the addition of convenience items like scope indicators i.e. parentheses). Given some ideology I_n, we call the language containing symbols only for its primitives and what can be defined using them (give or take a few scope indicators) L_{I_n} its perspicuous language. We can then say that one ideology is expressively equivalent to another just in case their perspicuous languages are expressively equivalent. We can extend this notion of a perspicuous language directly to a specific theory's ideology as follows: A language L_{T_i} is perspicuous for a theory T_i just in case it contains symbols only for the ideology that appears in T_i (give or take convenience items like scope indicators). We can now state the expressive power innocence criterion that I will be defending (more technically minded readers will find more precise discussion of expressive equivalence in Appendix A):

EXPRESSIVE POWER INNOCENCE CRITERION: some ideology I_k and some other ideology I_j are equally parsimonious if their perspicuous languages L_{I_j} and L_{I_k} are expressively equivalent.[3]

The expressive power innocence criterion gives us a sufficient condition for when arguments from ideological parsimony fail. They fail when the perspicuous language of T_i is expressively equivalent to the perspicuous language of T_j.[4]

[2] In this sense, the notion of expressiveness in play here is propositional and intensional. I do not mean to claim that all good notions of expressiveness are like this, only that this is the one I will be using.

[3] Nota Bene: This definition does make ideological parsimony relative to a class of models (as explained in Appendix A), but I think this is a harmless relativism. It is relative in the sense that anything that is relative to meaning is relative. In most cases we care about, there will be a clear 'right' class of models to use in making the comparison: namely, those we used when determining validities and analyticities for the languages. For example, for first order theories, the 'right' class of models will be models of predicate logic where the predicates of the theories stand in any common-ground analytic relations.

[4] As noted in fn. 3, expressive equivalence is always relative to the class of models used to give meaning to the language(s). So in order to use the criterion effectively, we will need to compare expressive power relative to the appropriate class of models. Spelling out which class of models is 'appropriate' will depend so much on the theories in question and the context of the debate that I doubt much can be said about it at this level of abstraction. But minimally, it should be a class of models for which a sensible semantics can be given for both languages, and where any common-ground validities and analyticities are respected.For example: if both theorists agree that 'bachelor' is equivalent to 'unmarried man,' then models including married bachelors are not appropriate.

3 The Argument from Accuracy

Now we can turn to the central argument of the paper: the argument from accuracy. The argument proceeds from a specific conception of the way in which epistemic virtues do their work and a specific theory of epistemic value to the conclusion that any adequate analysis of ideological parsimony must concur with the EXPRESSIVE POWER INNOCENCE CRITERION in all of its verdicts. I will not be able to offer full-blooded defenses of the theories of theoretical virtue and epistemic value that I will rely on, but these defenses can be found elsewhere. Instead, I will give a brief explanation of each before showing how they can be combined to produce an argument for the EXPRESSIVE POWER INNOCENCE CRITERION.

3.1 Virtue-Probabilism

Arguments from theoretical virtue attempt to show that one theory is better than its rival(s), even though all theories are consistent with the data.[5] We can divide theoretical virtues between *epistemic* and *pragmatic* virtues. Epistemic virtues provide epistemic reasons in favor of the theories that possess them. Following Ernest Sosa, I define an epistemic reason as a reason to affirm in the effort to be right, reliably enough.[6] Pragmatic virtues provide reasons to use a theory, truth be damned. I have nothing against pragmatic virtues, but I am only interested in ideological parsimony as an epistemic virtue.[7] Thus, I will argue that any analysis of ideological parsimony that violates the EXPRESSIVE POWER INNOCENCE CRITERION is inconsistent with regarding ideological parsimony as an epistemic virtue.

Epistemic likelihood is given by probability functions. There are a number of arguments for this conclusion, which I won't rehearse here.[8] But an important feature of probability functions is that they assign equal values to logically equivalent theories. Thus, if two theories are logically equivalent, one cannot be more likely than the other.[9] This gives us our first premise in the argument from accuracy.

FIRST PREMISE: Logically equivalent theories are equally likely to be true.

3.2 The Value of Truth

Epistemic virtues are the ones that make a theory more likely to be true. They are characteristic of theories we affirm in the effort to be right, reliably enough. This makes truth the only value that epistemic virtues are sensitive to and gives us our next premise.

SECOND PREMISE: Theories that are equally likely to be true are equally epistemically virtuous.[10]

Although I will not be giving a full defense of this premise, it is necessary to say a bit more about epistemic value. In his *Writing the Book of the World*, Ted Sider has proposed that the use of joint-carving ideology is itself epistemically valuable.[11] He makes this point by comparing 'regular' color predicates with 'grueified' ones. The color of the world's supply of emeralds and sapphires may be truly described using

[5] What exactly 'the data' amounts to and what it takes to be consistent with it is a difficult matter for another day.

[6] Sosa [2015].

[7] This does set me against those, like van Frassen [1989], who argue that all theoretical virtues are pragmatic. But that is a fight for another day.

[8] But see Cox [1946], Joyce [1998], [2009], Easwaran and Fitelson [2015], Pettigrew [2016], Leitgeb and Pettigrew [2010a], [2010b], Maher [1997], Christenen [1996].

[9] This places my view of the inferential role of theoretical virtues in the same company as Bayesian approaches. I only need probabilism, not full blown Bayesianism, to make the argument work, but arguments for Bayesianism will be arguments for the premise I need. See McGrew [2003] and Climenhaga [2017] for a Bayesian case; Douven [2016] for opposition.

[10] Starting with Goldman, there is a tradition in epistemic value theory called *veritism* that takes truth to be the sole epistemic virtue. I don't quite need full blown veritism here; it is sufficient for my purposes that theories that are equally likely to be true are equally virtuous. But arguments for veritism are arguments for my premise. See Goldman [1999]; see also Konek and Levinstein [Forthcoming] for the development of a veritist epistemic decision theory

[11] Sider [2011].

the familiar 'green' and 'blue,' or Nelson Goodman's 'bleen' and 'grue.'[12] But, he contends, the blue/green description is clearly better. This betterness is explained by blue and green being more fundamental concepts (or more structural, as he puts it) than grue and bleen. He later argues that it is epistemically better to know more fundamental (or structural) truths, a kind of epistemic goodness that cannot be explained in veritistic terms.[13]

With Sider, I agree that some concepts carve more natural joints than others. And, as one interested in truth, I agree that it is good to know which concepts those are. But I do not think that the superiority of the blue/green description is explicable in terms of epistemic value over and above the value of truth. Rather, I think the added value is a pragmatic matter. We think in terms of blue and green. It is therefore less of a cognitive strain for us to use the blue/green description. If a community naturally thought in terms of grue/bleen, they would be right to say that the grue/bleen description is better. It may be that we are right and they are wrong (or that they are right and we are wrong) about which concepts are more natural, but I maintain that even if we were to learn that grue and bleen carved closer to the joints in nature than blue and green do, we would be right to maintain our verdict that the blue/green description is superior for us.

3.3 Bringing It All Together

The argument from accuracy will show that any adequate analysis of ideological parsimony as an epistemic virtue must respect the EXPRESSIVE POWER INNOCENCE CRITERION. The argument itself is fairly straightforward.

We start with a theorem of the probability calculus. If $\phi \leftrightarrow \psi$ is valid (that is, if ϕ and ψ are logically equivalent), then $\Pr(\phi) = \Pr(\psi)$. Thus, if ϕ is more likely to be true than ψ is, then $\phi \leftrightarrow \psi$ is false in some model(s). Consequently, if T_1 is more epistemically virtuous than T_2, then T_1 and T_2 are not logically equivalent. Epistemic virtues do not divide logical equivalents.[14]

Any attempt to precisify ideological parsimony that does not respect the EXPRESSIVE POWER INNOCENCE CRITERION will divide logical equivalents. Suppose two collections of ideology, I_1 and I_2, are expressively equivalent, but some proposed criterion of ideological parsimony deems I_1 more parsimonious than I_2. Now take a theory T_1 whose ideology just is I_1. By assumption, there exists a function $Tr()$ from the perspicuous languages for I_1 to that of I_2. But this means that there exists a theory, $Tr(T_1)$, whose ideology just is I_2 and is logically equivalent to T_1. So the proposed criterion divides logical equivalents, and consequently cannot be the analysis of an epistemic virtue. Although the EXPRESSIVE POWER INNOCENCE CRITERION does not aspire to analyze ideological parsimony, it does create a necessary condition for any analysis of ideological parsimony that could be the analysis of an epistemic virtue.[15]

The argument from accuracy is the primary reason to accept the EXPRESSIVE POWER INNOCENCE CRITERION. We shall now turn to consequences of accepting it and objections that might be launched against it. We begin by dispatching rival approaches to ideological parsimony and then turn to use the EXPRESSIVE POWER INNOCENCE CRITERION to assess arguments from the literature.

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[12] Reminder: something is grue iff it is observed before 1/1/2028 and it is green, or it is not observed before 1/1/2028 and it is blue; something is bleen iff it is observed before 1/1/2028 and it is blue, or it is not observed before 1/1/2028 and it is green.

[13] Sider [2011] section 4.2.

[14] A wrinkle: we may be uncertain whether two ideologies are expressively equivalent. If so, then we may still be rational in assigning different probabilities to logically equivalent theories; Bayesian superbabies may be logically omniscient, but we aren't. This is an instance of the problem of logical learning. The best framework for modeling logical learning is Garrabrant et al.'s [Ms] "Logical Induction," in which the probabilities of logical equivalents converge in the limit. This is enough for the argument. As we learn more logic, we approximate the Bayesian ideal. Once we know which theories are equivalent, we assign them equal probabilities.

[15] My thanks to Veronica Gomez for pointing out this feature of EXPRESSIVE POWER INNOCENCE CRITERION to me.

4 Rivals

There are several proposed criteria for ideological parsimony in the literature. For now, I am going to evaluate them as attempts to analyze an epistemic virtue (see §5.5 for an alternative interpretation). So analyzed, all of them fall prey to the argument from accuracy. As we have seen, if a proposed precisification of theoretical virtue divides logically equivalent theories, then it is not the precisification of an epistemic virtues. The best way to test this is to compare minimal pairs: theories that are as similar as possible while differing with respect to the proposed virtue. I will use this minimal pair test to show that some intuitive precisifications of ideological parsimony that conflict with the expressive power innocence criterion fail; whatever goodness they capture, it is not epistemic.

We can use this test to eliminate rival precisifications of ideological parsimony that are somewhat intuitive and conflict with the expressive power innocence criterion. In particular, we can eliminate the COUNTING CRITERION, the KINDS COUNTING CRITERION, and the MERE DELETION criterion.[16] We will do this by providing logicaly equivalent theories that satisfy each, showing that ideological parsimony so-precisified is not an epistemic virtue. But first, we state the criteria. In doing so, it is important to recall that I am considering (and arguing against) these as guides to truth, criteria that makes theories that have them more probable than otherwise. There might be other interpretations (as non-epistemic virtues) of these same parsimony principles that are beyond the scope of my arguments, see §5.5:

THE COUNTING CRITERION: Some ideology I_j is more parsimonious than some other ideology I_k if I_j has fewer bits of ideology than I_k.

THE MERE DELETION CRITERION: If I_j is obtained from I_k by deleting some bit of ideology, then I_j is more parsimonious than I_k.

THE KINDS COUNTING CRITERION: Some ideology I_j is more parsimonious than some other ideology I_k if I_j has fewer kinds[17] of ideology than I_k.

These are the criteria. Now for the theories. We begin with a pair that takes down both COUNTING CRITERION and the MERE DELETION CRITERION. The first is a mereological theory with parthood taken as primitive. The second is a mereological theory with both parthood and overlap taken as primitive. Since they both have a finite ideology, the first theory satisifies both the COUNTING CRITERION and the MERE DELETION CRITERION relative to the second. But since both are axiomatizations of classical mereology, they are logically equivalent when the standard interdefinition of parthood and overlap is added (or: in the class of models where the interdefinition of parthood and overlap is a theorem). This shows that both criteria cut too finely to be epistemic virtues. Note that both unrestricted fusion axioms are actually axiom schemata, which we are using to avoid adding plural quantification to the ideology.

MEREOLOGICAL THEORY ONE:[18]

i. All predicate logic tautologies — TAUT

ii. AXIOM: $\forall x Pxx$ — PART REFLEXIVITY

iii. AXIOM: $\forall x \forall y((Pxy \wedge Pyx) \rightarrow x = y)$ — ANTISYMMETRY

iv. AXIOM: $\forall x \forall y \forall z((Pxy \wedge Pyz) \rightarrow Pxz)$ — TRANSITIVITY

[16] Although it tends to be the initial heuristic used, I've yet to encounter anyone actually accepts the counting criterion; arguments against it may be found in Cowling [2013], Sider [2013], and Goodman [1951]; Sider [2013] employs the mere deletion criterion for fundamental theories, while Cowling [2013] proposes the kind counting criterion].

[17] Cowling never defines a 'kind' of ideology, but examples include: modal operators (of all sorts), truth-functional connectives, and mereological predicates.

[18] Courtesy of Varzi and Cotnoir [ms.]

v. AXIOM: $\forall x \forall y(\neg Pxy \rightarrow \exists z \forall w(Pwz \leftrightarrow (Pwx \wedge \neg \exists t(Ptw \wedge Pty))))$ REMAINDER

vi. AXIOM: $\exists x\varphi \rightarrow \exists z(\forall y(\varphi \rightarrow Pyz) \wedge \forall w \forall y((\varphi \rightarrow Pyw) \rightarrow Pzw))$ UNRESTRICTED FUSION

MEREOLOGICAL THEORY TWO:

i All predicate logic tautologies TAUT

ii AXIOM: $\forall x Oxx$ OVERLAP REFLEXIVITY

iii AXIOM: $\forall x \forall y((Pxy \wedge Pyx) \rightarrow x = y)$ ANTISYMMETRY

iv AXIOM: $\forall x \forall y \forall z((Pxy \wedge Pyz) \rightarrow Pxz)$ TRANSITIVITY

v AXIOM: $\forall x \forall y(\neg Pxy \rightarrow \exists z \forall w(Pwz \leftrightarrow (Pwx \wedge \neg \exists t(Ptw \wedge Pty))))$ REMAINDER

vi AXIOM: $\exists x\varphi \rightarrow \exists z(\forall y(\varphi \rightarrow Pyz) \wedge \forall w \forall y((\varphi \rightarrow Pyw) \rightarrow Pzw))$ UNRESTRICTED FUSION

As we can see, THEORY ONE is an axiomatization of classical mereology using the ideology of quantificational logic with identity and a primitive parthood predicate, while THEORY TWO is an axiomatization of classical mereology using the ideology of quantificational logic with identity plus both primitive parthood and primitive overlap predicates. But since they are both axiomatizations of classical mereology, they are logically equivalent in models where P and O take their intended interpretations (that is: models where $\forall x \forall y(Pxy \leftrightarrow \forall z(Ozx \rightarrow Ozy))$ is true; adding this definition to PART REFLEXIVITY allows us to derive OVERLAP REFLEXIVITY and *vice-versa*). But the ideology of THEORY ONE is more parsimonious than the ideology of THEORY TWO by both the COUNTING CRITERION and the MERE DELETION criterion. Both have a finite ideology, and that of THEORY ONE may be obtained from THEORY TWO by deleting the 'Overlap' relation. But since classical mereology doesn't get any more likely when reaxiomatized with different primitives, neither COUNTING CRITERION nor MERE DELETION CRITERION are the right way to precisify ideological parsimony as an epistemic virtue.

It will require a different example to show how the KIND COUNTING CRITERION fails. Here we must be a bit less precise, because we do not have a rigorous definition of an ideological kind on hand. In what follows, however, I will rely on only two claims: (i) adding a modal operator to a truth-functional propositional language adds a new kind of ideology, and (ii) 'truth-function' is a kind of ideology. Since the primary defender of the KIND COUNTING CRITERION endorses (i)[19], that leaves (ii) as the only risky commitment I must make. But (ii) strikes me as fairly low risk. Insofar as I have any intuitive grasp on the notion of an ideological kind (and thus am willing to countenance them sans definition), truth-functions form one. With (i) and (ii) in hand, we can use the translations between intuitionistic propositional logic and the modal propositional system S4 to generate an infinity of counterexamples to the KIND COUNTING CRITERION.[20]

The language of intuitionistic propositional logic contains proposition letters and truth-functional connectives.[21] The language of (basic) modal logic adds a modal operator to its base of propositional variables and truth-functional connectives. Thus, it contains more kinds of ideology than intuitionistic propositional logic. But since the two languages are expressively equivalent, any S4-theory is logically equivalent to one in the language of intuitionistic propositional logic. The KIND COUNTING CRITERION divides many logical equivalents.

[19] Cowling [2013].

[20] See Godel [1933] and McKinsey and Tarski [1948] for the proofs. For those surprised by this translatability, it will be helpful to recall a few facts. First: the basic idea behind intuitionist logic is provability. The intuitionist only accepts theorems that she has a constructive proof of. Thus, in her mouth, $\neg P$ means 'there is no constructive proof of P.' This is why she rejects excluded middle: there is a third option between P being true and there being no constructive proof that P: namely, that P is both true and lacking a constructive proof. Second: there is a provability interpretation of the modal logic formalism, where $\Box P$ means 'it is provable that P' and $\diamond$ P means 'it is consistent that P.' Concatenating the provability-interpreted box with classical negation yields a meaning of 'it is not provable that P' for the expression $\neg\Box P$, which is not that far from the meaning of intuitionistic negation. This is the central insight of both the Godel and the Tarski-McKinsey constructions.

[21] It requires an infinity of truth-values to characterize intuitionistic logic. See Kleene [1937].

5 Extant Parsimony Arguments

Next we will look at some consequences of adopting EXPRESSIVE POWER INNOCENCE CRITERION for several first-order metaphysical debates. Examining some arguments from ideological parsimony in the literature, we will see if they pass muster. If the EXPRESSIVE POWER INNOCENCE CRITERION deems an argument that looks unsuccessful unsuccessful, or fails to deem an argument that seems good unsuccessful, that is to its credit.

We will examine the following cases: Ted Sider's mereological nihilism vs. its traditional rivals, modal theories with primitive actuality vs. modal theories without it, David Lewis's attempt to reduce modality to quantification over worlds, and an argument against the "moving spotlight" theory of time.

5.1 Nihilism Old and New

Very generally, mereological nihilists deny that anything is a part of anything other than itself. Traditionally, this means replacing UNRESTRICTED FUSION with an axiom that denies the existence of proper parts. But traditional nihilists do not dispute axioms like TRANSITIVITY and REFLEXIVITY. Parts may well have these properties, if there were any. Thus, a traditional nihilist mereology has a very similar axiomatization to a traditional universalist mereology, with changes only to the axioms that say which composites exist. We give one below:

i All predicate logic tautologies TAUT

ii AXIOM: $\forall x Pxx$ REFLEXIVITY

iii AXIOM: $\forall x \forall y \forall z ((Pxy \wedge Pyz) \rightarrow Pxz)$ TRANSITIVITY

iv AXIOM: $\forall x \forall y (Pxy \rightarrow x = y)$ NIHILISM

Ted Sider's mereological nihilism departs from this tradition.[22] Sider argues for the wholesale elimination of mereological ideology. Thus, he does not simply claim that nothing has a proper part; he claims that 'proper part' has no place in a fundamental theory of the world. While this has a similar effect on his ontology - it lacks composites - as traditional nihilism would, it changes his mereological theory and therefore his ideology considerably. The traditional nihilist theory above has the same ideology as the universalist's; where it differs is in ontology, in the things it claims exist. Sider's mereological theory, by contrast, omits all of the distinctively mereological ideology and the consequently the axioms formulated in it. We give it below:

i All predicate logic tautologies TAUT

This is intuitively an advance in ideological parsimony. Sider's ideology, lacking the ability to say anything about parts, seems simpler than that of a theory which has lots to say about what parts would be like, were there to be any.

Since Sider's ideology is only the bare logical vocabulary of predicate logic, it is expressively weaker than that of the traditional nihilist, lacking any equivalent way to express the 'parthood' predicate. Consequently, our judgement about its relative simplicity agrees with the EXPRESSIVE POWER INNOCENCE CRITERION. The addition of a 'parthood' predicate added expressive power and so was not deemed ideologically innocent.

5.2 Modal Theories and Actuality

We can find a second test case in modal metaphysics. One of the longest-running disputes among modal metaphysicians has been over the status of actuality. Reductionists like David Lewis[23] and Robert

[22] Sider [2013].
[23] Lewis [1986]

Adams[24] explain actuality in terms of something else (indexicality, truth) while primitivists such as Phillip Bricker[25] take actuality as a simple, unanalyzed property of worlds. There is, *prima facie*, an argument from ideological parsimony in favor of the reductionists. The EXPRESSIVE POWER INNOCENCE CRITERION does not stand in its way.

For simplicity, let us imagine two modal metaphysicians who disagree only about the status of actuality. They agree on which modal logic is correct, and they agree about what's possible/necessary. Thus, we can may describe their theories as below:

MODAL THEORY ONE

i All S5 tautologies TAUT

ii Possibility postulates POSS

MODAL THEORY TWO

i All S5 tautologies TAUT

ii Possibility postulates POSS

iii Hybrid Axioms HYBRID

iv Actuality Postulates ACT

The first theory includes axioms for modal logic and axioms for which things are possible (necessities may be derived from these two). I will assume these can all be described in the langauge of propositional modal logic (nothing much turns on whether the language is propositional or first order, so we can count this as a simplifying assumption). The second theory includes the same modal logic and theory of possibility, but it adds in axioms for the logical behavior of an actuality operator and a substantive theory of what's actual. The language of this theory is thus a simple hybrid language: that of propositional modal logic enriched by an actuality operator.

It seems fairly clear that the ideology of MODAL THEORY TWO is more complicated than that of MODAL THEORY ONE. Aside from additional postulates, the actuality operator requires new axioms to describe its logical behavior. And the EXPRESSIVE POWER INNOCENCE CRITERION agrees. It is a well-known result that simple hybrid logic is expressively superior to simple modal logic, both propositional and first-order.[26] Thus, the addition of an actuality operator to the ideology of MODAL THEORY ONE is not innocent. As before, we have a clear case where intuition and EXPRESSIVE POWER INNOCENCE CRITERION are in harmony.

5.3 Modal Reduction and Quantification

In both the metaphysics of modality and the metaphysics of time, we find debates between those who prefer primitive operators such as 'possibly' and 'was' and those who prefer to reduce those operators to quantification over worlds/times. A typical telling of the situation goes something like this: reductionist purchase an improvement in ideological parsimony in the coin of ontology; primitivists have a leaner ontology but at the price of a bloated ideology.

The EXPRESSIVE POWER INNOCENCE CRITERION calls this telling into question. Basic modal and tense logics are provably expressively weaker than two-sorted predicate logics with quantification over worlds/times.[27] Furthermore, proposals to increase the expressive power of modal/tense logics to capture some of the things that can be done with quantification over worlds/times do not generally achieve full

[24] Adams [1974]

[25] Bricker [2006].

[26] see Kocurek [forthcoming] for the first order case, Areces and ten Cate [2007] for propositional.

[27] See Blackburn and von Bentham [2006], Hodkinson and Reynolds [2006].

expressive equivalence, and I know of no proposal that would achieve expressive superiority. Consequently, the EXPRESSIVE POWER INNOCENCE CRITERION says that there is no successful argument from ideological parsimony for reductionism about time/modality over primitivism. This places the primitivist in good position as regards parsimony over the reductionist. By most accounts of ontological parsimony, her view is more parsimonious. By any adequate account of ideological parsimony, her view is at least not less ideologically parsimonious. So her view is probably more parsimonious and at least not less parsimonious than that of the reductionist. If theoretical economy is the main motivation for reductionism, this leaves the reductionist in an awkward position. It does not, of course, constitute anything like a conclusive argument for primitism. Other virtues must have their say. But it leaves the reductionist with more work to do than is generally thought. It is also a surprising and interesting result of accepting the EXPRESIVE POWER INNOCENCE CRITERION and a good case of it helping us make progress in first order metaphysical disputes.

5.4 The Moving Spotlight

Another argument from ideological parsimony may be found in the philosophy of time, targeting "moving spotlight" theories of time. This argument does not fail, but its non-failure is interesting. It will turn out that some of the moving spotlighter's alleged excess machinery is innocent, but some of it is not.

Moving spotlight theories address two debates in the philosophy of time (among other things). The first is over ontology. Presentists think that only the present time and present things (give or take a few abstracta) exist. Their opponents, eternalists, think that all times - past, present, future - and all of the things - past, present, future - exist and are on an ontological par. As a result, presentists and eternalists account for truths about the past and future in very different ways. In order to see how, we'll start with a fairly ordinary, present-tense truth: the sun is shining. This is true just in case there is a sun, and that sun shines at the time of utterance (which for some eternalists may be the value of a covert variable in the expression's logical form, while for presentists it will be picked out by the tense indexical NOW which is implicitly part of the English verb 'is'). On that, everyone agrees. For eternalists, truths about the past and future are very much like this truth. For an eternalist, 'the sun was shining' will be true just in case, at a time earlier than ours, there exists a sun and that sun is shining. She can do this because she believes that all times - past, present, future - and all things - past, present, future - exist, just as much as we and our shining sun do now. A presentist has to say something different, since she does not think that past times and past things, or future times and future things, exist. So she introduces tense operators. A tense operator attaches itself to a tenseless sentence (or sentence fragment, since English sentences come with tense baked in) and commands us to evaluate its truth not at the present but the past or the future. Thus, a sentence like 'the sun was shining' becomes PAST (sun shine), true just in case the sun was shining in the past. This allows the presentist to state facts about the past (or future) without an existing past or future. In order to rid herself of that ontology, she has adopted the ideology of tense operators.

The second debate is over change. A-theorists believe that change of some sort is a deep feature of the world, and this is ultimately to be accounted for by change from past to present to future, making the distinction between past, present, and future an important feature of the world. Their opponents, B-theorists, think that change is shallow. Change for a B-theorist is merely difference along a time-like dimension. For example, according to a B-theorist, all it is for me to change from sitting to standing is for a past version (I use 'version' here to remain neutral between stage theories and worm theories) of me to be sitting while a present one is standing. According to an A-theorist, however, there is only one version of me, which once sat and now stands.

The moving spotlighter combines an eternalist ontology with an A-theorist's approach to change, but only for a few very special properties. Like the eternalist, the moving spotlighter places all times on a par: past, present, and future. And like the B-theorist, she accounts for most change by difference along a time-like dimension. I change from sitting to standing by having a sitting version succeeded by a standing version. But she adds something: a few special properties that change in the A-theorists' sense. The tense properties. This is what gives the view its name. The moving spotlighter believes in an objective present, a primitive property of 'being present' that is true of one time and then another. This property "lights up" each time in succession, making it the case that what is past, present and future changes not simply by being arranged in an 'earlier than' relation, but by being before, at, or after a time that is objectively present. Thus, the moving spotlighter supplements the B-theorists' quantification over times with tense operators that track the movement of the spotlight (change in the facts about the objective present).

Sider has questioned whether the small amount of "genuine" or A-theory-ish change that the moving spotlighter manages to secure is worth the cost of the extra ideology - the addition of tense operators and the presentness property.[28] But the tense operators, it turns out, are free. Quantified tense logic is expressively equivalent to a fragment of two-sorted quantificational logic, and there is no known way to extend it to the entire language.[29] However, the same cannot be said for the 'is present' predicate. Just as the Siderian nihilist can remove the mereological predicates and achieve an expressive decrease against the regular mereologist, the eternalist can dispense with the 'is present' predicate and achieve an expressive decrease against the moving spotlighter.

5.5 A Fundamentality Interpretation

Before we depart our discussion of the impact of the EXPRESSIVE POWER INNOCENCE CRITERION on the literature, it is good to consider another interpretation of parsimony arguments to make clear the scope of the critique. So far, I have explored parsimony arguments as arguments for the truth of a theory. But it might be used in other ways; it may be used only as a guide to the fundamentality (or fundamental truth) of a theory. According to this version of the parsimony principle, more parsimonious theories need not be more likely to be true, but are more likely to be fundamental. Two theories may both be true, but the fact that one is more parsimonious gives us reason to think that its ideology is more fundamental. This allows rivals to the EXPRESSIVE POWER INNOCENCE CRITERION to avoid the argument from accuracy, because it is possible to have two logically equivalent theories, one of which is more fundamental than the other.[30]

I think there is something to the fundamentality interpretation, so I will not attempt to argue that it is not a route to viable parsimony arguments. But I want to be clear on its possible justification. Attempts to justify the principle of parsimony have varied from brute intuition, theological arguments about the kind of world a creator-god would make, and appeals to scientific practice or the history of science.[31] I don't think it intuitive *a priori* that the world should be parsimonious, and I find the theological arguments dubious, so I think the best of these justifications is from the history of science. There seems to be a property of theories - call it parsimony or simplicity - that better theories tend to have. The catch for the fundamentality interpretation: in key motivating cases, the theories being compared are a true theory and a false one (or a theory that is closer to true and one that is further from it), not a more fundamental and a less fundamental theory.

The prime example of a simpler theory triumphing over a more complex one comes from the debate between Ptolemaic geocentric theories of the solar system and Copernican heliocentric ones. When Copernicus first advanced his theories, he did not have any observation that his system accounted for but his Ptolemaic rivals could not. The two rivals were observationally equivalent. Instead, Copernicus was able to account for the observations with a simpler model, using fewer free parameters and explaining things that Ptolemy could not. This ultimately marked his theory as more correct than his rivals'. If the scientific virtue we are latching onto with our parsimony talk is one that tracks fundamentality, then the heliocentism vs. geocentrism case cannot be brought out in its justification. Ptolemaic theories were not even derivatively true (or somehow derivable from the also-false Copernican theory). The stakes in this debate were not fundamentality, but truth.

[28] Sider 2011

[29] Hodkinson and Reynolds [2006]

[30] In personal correspondence, Sider has indicated that this is how his use of the MERE DELETION CRITERION in his [2013] is intended.

[31] Sober [2015] Ch. 1 has a good summary.

But there are some cases from the history of science that might be more favorable to the fundamentality interpretation. Sider discusses the case of Newtonian vs. Galilean spacetime. According to Newtonian theories of spacetime, space and time are absolute, and therefore there is absolute position and motion. By contrast, Galilean spacetime does not accept absolute position or motion. Everything is relative to an inertial frame. Unlike geocentric and heliocentric models of the solar system, Newtonian and Galilean models of spacetime do not contradict each other. Newtonian models add an absolute coordinate system to Galilean models. But it could be that what is fundamental is only what we find in the Galilean theory, while some convention fixes a preferred frame that gives rise to the in-some-sense-absolute spacetime of the Newtonian theory. In this case, parsimony would be a guide to fundamentality (although in fact there is no such convention).

It would require a more exhaustive examination of the history of parsimony arguments in the sciences than I have space to undertake to settle whether they tend to be guides to fundamentality rather than guides to truth. For now, I am content to acknowledge the limits of my arguments and register some skepticism that the fundamentality interpretation ultimately will be the one the history of science ends up vindicating.

6 Objections

Finally, we will consider two objections. The first comes from a special class of languages: those with an 'is primitive' operator.[32] The ability to include axioms in a theory that say what the theory takes as primitive poses a threat to the claims of equivalence that drive the argument from accuracy. I respond by arguing that a theory is not equivalent to a cousin theory with the same axioms but an additional one that says what the theory's primitives are. The second comes from Nelson Goodman, who considered an expressive power analysis of parsimony but rejected it as undermining the goal of seeking a parsimonious set of primitives out of which to build a theory's ideology. I respond by showing that Goodman and I have incompatible conceptions of the role of parsimony as a theoretical virtue. Goodman's is pragmatic while mine is not.

6.1 Higher-Order Languages With a 'Primitive' Operator

So far, we have been working with cases where the perspicuous languages are fairly well understood, with a wide body of model-theoretic results to draw on. Now we will consider theories formulated in languages that are less well-studied, but seem fairly natural for our purposes: languages where we are allowed to list our primitives in the object-language. In these languages, we introduce an explicit 'is primitive' operator $\mathfrak{p}$, which allows us to give a list of symbols which stand for the ideological primitives in a theory. A theory in a language like this can (perhaps must?) then have a 'primitives axiom,' that is: an axiom which lists the theory's primitives. Any language can be extended with a 'primitive' operator, and so any theory can be supplemented with a primitives axiom. If we are required to list our primitives in a new axiom, this poses a threat to the argument I've given for the EXPRESSIVE POWER INNOCENCE CRITERION. Languages that merely allow us to list our primitives aren't a problem. But if forced to include a primitives axiom, mereological theories 1 and 2 are no longer logically equivalent, and thus the argument from accuracy will no longer apply. Indeed, any two theories that differ in their primitives will no longer be logically equivalent. This puts rivals to the expressive power approach back in play.

My response takes the form of a challenge: "Sez Who?" Why must our theories specify their primitives in the object-language? It will unquestionably be true about our theories that they employ such-and-so primitives. But there are lots of things that will be true about our theories that we do not need to say in the object-language, or add special axioms that break natural equivalences to account for. To take a somewhat absurd example: every theory is stated in a certain number of characters. So there will be

[32] Something similar to the operator could be accomplished with an 'is primitive' predicate. What I say about the languages with the operator applies *mutatis mutandis* to those with the predicate.

some truth about each theory which says how many characters are in its statement. But it would be a bit absurd to insist that our theorizing take place in a language with apparatus designed to talk about the character-count of theories, so that each theory is supplemented with a 'character-count axiom,' thereby breaking logical equivalences. What makes a 'primitives axiom' different from a 'character-count' axiom, such that we ought to include one in our theories?

I can think of two potentially promising responses to the "Sez Who" challenge. First: the Closure Response. The closure response answers the challenge with: logic. It's fairly standard to think of theories as closed under logical consequence. If, therefore, it turned out that every theory in a language that includes a $\mathfrak{p}$-operator, as a matter of logical closure, includes a primitives axiom, then it looks like the only thing stopping us from specifying our primitives in our theories is our refusal to include a $\mathfrak{p}$-operator in our language. This refusal on the part of the EXPRESSIVE POWER INNOCENCE CRITERION defender then comes to look awfully convenient, and somewhat arbitrary.

Second: the Equivalence Response. The equivalence response answers the challenge with: because a theory is always equivalent to its expansion with a primitives axiom. Inquiry into the conditions under which two theories are equivalent is ongoing. But there is a fair case to be made that it is something less stringent than logical equivalence. If so, then this leaves open the possibility that, under a plausible and well-defined notion of theoretical equivalence, a theory and its expansion with a primitives axiom are equivalent. 'Because they are equivalent' seems like a good reason to replace a theory with its primitives-axiom-supplemented twin.

I respond to these with a dilemma. We can partition the space of possible logical relationships between a theory and its primitives-axiom-enhanced cousin as follow: (i) the two theories could be logically equivalent, (ii) the enhanced theory could be logically stronger, (iii) the unenhanced theory could be logically stronger, or (iv) they could be logically incomparable. We can rule out (iii) and (iv) pretty easily. In order to compare the two at all, we must use the language of the enhanced theory and its attendant consequence relation. If we were to use the language of the unenhanced theory, the enhanced theory would be gibberish. Since the enhanced theory is a superset of the unenhanced theory, it will entail the unenhanced theory according to any sane consequence relation. That leaves two options: the enhanced theory is logically stronger, or the two theories are logically equivalent.

If both theories are logically equivalent, then the objection fails. Logical equivalence is transitive. So if T_1 and T_2 are equivalent, and the enhanced versions of T_1 and T_2 are logically equivalent to T_1 and T_2 respectively, then the enhanced versions of T_1 and T_2 will be logically equivalent to each other. Thus, if the objection is to work at all, the enhanced theory must be logically stronger than the unenhanced version. This undermines the closure response. A theory must be logically equivalent to its deductive closure, because by definition it entails its own deductive closure.

That leaves only the equivalence response. There are approaches to theoretical equivalence that allow for theories that are not logically equivalent to be theoretically equivalent nonetheless. We find several proposed formal criteria of theoretical equivalence in the literature.[33] This opens the door for a theory and its primitives-axiom-supplemented twin to be deemed theoretically equivalent despite not being logically equivalent.

Ideally, we would be able to test for this mathematically. But two things prevent us from doing that. First: there is no standard formal criterion for theoretical equivalence. That question remains open. Second: we do not have a good semantics for the $\mathfrak{p}$-operator. Fortunately, we can introduce some non-formal considerations that push us toward rejecting the claim of equivalence.

Which primitive concepts are the correct ones is a fact about the world. A theory can be true while taking things as primitive that should not be so taken.[34] If this is true, then it looks like a primitives axiom is a genuine addition to a theory. It says more about the world than the theory itself does. And if that is true, it tells against regarding the theories as equivalent. One says more than the other. Absent a powerful reason - such as the verdict of a well-regarded formal criterion of theoretical equivalence - to think

[33] See Barrett and Halverson [2016] for a good summary.
[34] Sider [2011] presents a compelling case for these theses.

otherwise, this should be enough to reject the equivalence response. Circumstances in which equivalent theories are not logically equivalent are very specific. We should not just suppose ourselves to be in them.

Of course, nothing I have said here *disallows* a language from having an 'is primitive' operator and theories from having a primitives axiom. But if those theories neither follow from nor are equivalent to their primitives-axiom-lacking cousins, the argument from accuracy still stands. Suppose we are interested in what's fundamental and want theories that list their primitives. Presumably equivalences among those will be much more rarer. But when they do occur, the EXPRESSIVE POWER INNOCENCE CRITERION will still apply.

6.2 Goodman's Objection

Nelson Goodman considered a similar approach to simplicity in *The Structure of Appearance*.[35] He proposed and rejected something more ambitious than the EXPRESSIVE POWER INNOCENCE CRITERION: he considered an analysis of simplicity in terms of expressive power, where one theory is simpler than another if the language of its ideology is expressively weaker. However, he rejected this analysis by observing that it provided no motivation for replacing defined terms with primitives. Since, he argued, the main goal of finding a simpler theory was to find primitives that could be used to define all desired predicates, the analysis of simplicity in terms of expressive power undercut the original motivation to seek simpler theories.

We can see how the objection works most clearly with an example. Consider the following two sets of primitives for a modal language.

MODAL PRIMITIVES ONE: $\Box, \diamond, \rightarrow, \wedge, \vee, \neg$, and a countable infinity of propositional variables.

MODAL PRIMITIVES TWO: $\Box, \rightarrow, \neg$ and a countable infinity of propositional variables.

All of the primitives in MODAL PRIMITIVES ONE can be defined out of primitives in MODAL PRIMITIVES TWO. Thus, if an expressive power analysis of ideological parsimony is right, the two sets of primitives are equally parsimonious. Likewise, if the EXPRESSIVE POWER INNOCENCE CRITERION is right, the extra primitives are ideologically innocent. And what goes for these goes for anything else definable from the primitives in MODAL PRIMITIVES TWO. We could add a primitive for all 16 bivalent truth functions and still not increase expressive power. And so, Goodman says, we might as well never define anything. Replacing a defined notion with a primitive one will never increase our expressive power.

If Goodman is right that the task of finding the simplest ideology for our theory is tantamount to finding the best base of primitives from which to define everything else, then this objection is fatal. But for reasons articulated in §3, he cannot be correct. Merely replacing primitives with defined notions that have the same truth conditions cannot make a theory more likely to be true, and so cannot be a way to increase ideological parsimony in the sense we are interested in–as an epistemic improvement. Of course, there are many contexts when reducing out number of primitives is useful. Two examples: first, when doing metatheory, it is often helpful to state our theory using the fewest kinds of lexical item (especially when things need to be proven by induction); second, sometimes we search for ways to define some terms in terms of others in order to find out which theories/languages are equivalent in the first place. But that usefulness is merely pragmatic. It's not the virtue we're looking for.

7 Conclusion

I have argued for a partial analysis of ideological parsimony, the EXPRESSIVE POWER INNOCENCE CRITERION. The criterion says that if the primitive ideology of one theory is expressively equivalent to that of another, then neither ideology is more parsimonious to the other. This has the consequence that additional ideology that does not increase expressive power is innocent. It can be added without taking

[35] Goodman [1951].

a hit to virtuousness. I have argued for this criterion in two ways. First: I have shown that popular criteria for ideological parsimony that conflict with it divide logically equivalent theories, and therefore doing better by their lights need not make a theory more likely to be true. Second: I have given some intuitive cases where one theory does have a simpler ideology than another, and in all of those cases the simpler theory was expressively weaker. Next, I considered two objections, both of which failed. I conclude that EXPRESSIVE POWER INNOCENCE CRITERION is true, and we are one step closer to an analysis of ideological parsimony.

A More Details on Expressive Equivalence

Here we will go into more detail on what I mean by expressive power and how it can be measured. Expressive power is relative to an assignment of meaning to the sentences of the languages, otherwise any old function will do. For the sake of specificity, I will assume that something like model-theoretic semantics is the right approach to meaning. This assumption is inessential to my project, but it has the twin virtues of specificity and conformity to the dominant paradigm. Much of what I say could be abstracted to other frameworks *mutatis mutandis*. A good semantic theory does at least two things: it gives us the truth-conditions for sentences of our language, and it correctly characterizes their logical behavior. In model-theoretic semantics, we do this by choosing a class of mathematical structures, called models, and then saying when a given sentence is true in a model (or some slight complication like model-point or model-assignment pair; we'll ignore such complexities for now).

Consequently, expressive equivalence is relative to choice of semantic theory. This just means that expressive equivalence is relative to fixing the meaning of the terms in the langauges. We fix the meaning with a class of models, rather than single intended model, because an important part of assigning a meaning is characterizing logical behavior. This isn't, in general, done with a single model because if we only had the one model to characterize the logical behavior of our terms, anything that is true in that model would be equivalent to the tautology and anything false to the contradiction, and we would end up with any number of equivalences that we don't want. The solution to unwanted equivalences is to add more models where the equivalences don't hold. So instead we start with a very general class of models (for a language with only truth-functions and propositional variables, for example, this class might be the class of all assignment functions) and then whittle it down so that the validities and other logical or semi-logical behaviors we want out of our language are respected (so if, for example, we intend P and Q to be contraries we omit any models that make both true; if we mean for P and Q to be equivalent, we omit models that assign them different truth values, etc.). But this whittling down will not, in general, leave only one model standing.

With this background, we can now say a bit more precisely how we will reckon expressive equivalence. Given a pair of languages L_1 and L_2, we require some class of mathematical structures $\mathbb{M}$ such that each $\mathfrak{M} \in \mathbb{M}$ is a model for both L_1 and L_2 (alternatively, sometimes we will not be able to give sensible semantics for both languages in terms of the same class of models, but we can do something just as good: some of the models in $\mathbb{M}$ can be used to give a semantics for L_1, others can be used to give a semantics for L_2, and there is a sensible/natural way to associate models of L_1 and models of L_2 that matches up models that are "the same"). We can then say that L_1 is expressively equivalent to L_2 just in case there exists a function Tr from sentences s_i of L_1 to sentences s_j of L_2 such that, for all models $\mathfrak{M} \in \mathbb{M}$, $\mathfrak{M} \models s_i$ iff $\mathfrak{M} \models Tr(s_i)$ (or, in the case where we could not give semantics for both languages with the same models but have a natural way of associating models with one with models of the other that are the same: for each model $\mathfrak{M} \in \mathbb{M}$ of L_1, $\mathfrak{M} \models s_i$ iff $\mathfrak{M}' \in \mathbb{M} \models Tr(s_i)$ where $\mathfrak{M}'$ is "the same" model as $\mathfrak{M}$ but is a model of L_2).

Informally, we have done the following. First: we have chosen models. If we are using different classes of model for each language, we have decided when two models count as "the same." Next, we have given semantics for our languages. These give us our truth-conditions and together with the models chosen characterize the logical behavior of all terms. Finally, we have checked to see if, for each sentence of the langauges, we can find one in the other language that is true under the exact same conditions and so exhibits the same logical behavior. If so, we have deemed those sentences co-expressive. If this search reveals a one to one correspondence between co-expressive sentences, we have expressively equivalent languages.

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